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Holographic Hado-Quarkonium

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Abstract

We consider the recently suggested model for some resonances near the open charm threshold as bound states of charmonium inside excited light mesons. It is argued in the soft-wall holographic model of QCD that such states of heavy quarkonium necessarily exist at sufficiently large spin of the light meson. The bound state is provided by the dilaton exchange through the 5D bulk. We also argue that the decay of such bound systems into mesons with open heavy flavors due to splitting of the heavy quarkonium can be treated as semiclassical tunneling and is suppressed. This behavior is in agreement with the known relative suppression of the decay of the discussed charmonium-like resonances into channels with D mesons.

1 Introduction

Recent observation¹ of resonances near the open charm threshold which decay into a specific charmonium state, J/ψ or $\psi(2S)$, and two or even one pion with no other observed conspicuous decay channels [4] suggests [2] that these resonances in fact contain the specific charmonium state ‘coated’ by an excited light-hadron matter. It has been further argued [5] that such hadro-charmonium systems are likely to arise due to binding of charmonium inside sufficiently excited light resonances through a QCD analog of the van der Waals interaction.

In this paper we investigate in some detail the interaction of heavy quarkonium with excited light mesons within the simplest soft wall model of holographic QCD [6] which provides linear behavior of Regge trajectories [7, 8] and a reasonable behavior of a heavy-quark potential [9]. In this approach the mesons can be considered as modes of the flavor gauge field in 5D theory (vector mesons) or a Wilson line of this gauge field along the fifth coordinate (π -meson). Large-spin mesons can be considered within this approach as well. We also have to introduce heavy quarkonium in this framework. It is clear that the corresponding heavy degrees of freedom are localized near the boundary at $z = 0$ at scales of order M_Q^{-1} , where M_Q is the mass of the heavy quark. In the leading approximation we shall substitute the heavy meson by the point-like source. The essential local operator on the boundary can be deduced from the physical arguments and is approximated by the trace of the stress tensor. Since this operator interacts with the dilaton we have a point-like dilaton source in the 5D theory. On the other hand the light excited meson with large spin is mainly localized on the IR wall scale [10]. Although it is better described by a long open string, this approximation is also reliable. Thus the appropriate configuration consists of a local source at the boundary $z = 0$ and extended meson at the z_{IR} interacting via dilaton exchange. It is shown that the interaction is strong enough to yield the bound state at a sufficiently large spin S of the light meson.

Another feature which has to be explained is the phenomenologically apparent suppression of the decays of hadro-quarkonium into states with pairs of heavy mesons with open flavor. In the discussed picture such decays would correspond to dissociation of heavy quarkonium due to forces from light degrees of freedom. We argue that in the limit of large heavy quark mass M_Q such dissociation can be described by semiclassical tunneling and is suppressed by the factor $\exp(-const \cdot M_Q^{1/2}/\Lambda_{QCD}^{1/2})$. Within the holographic approach this

¹Recent reviews can be found in Refs. [1, 2, 3].

process is a generalization of the picture discussed in [11]. Clearly, the suppression of the dissociative decays is in qualitative agreement with the available observations.

The subsequent material in the paper is organized as follows. In Sec.2 a description of the interaction of heavy quarkonium with soft gluonic field is recapitulated and formulated in terms of the trace of the stress tensor in low-energy QCD. In Sec.3 the interaction of the heavy quarkonium with light meson is described in terms of the soft wall holographic model and in Sec.4 it is shown that this interaction necessarily results in existence of bound states for sufficiently high orbital excitation of the light meson. In Sec.5 we consider the dissociation process for decays of the bound states into heavy meson pairs with open flavor and argue that such process generally carries a semiclassical suppression at large mass of the heavy quark. Finally, in Sec.6 we summarize our conclusions.

2 Quarkonium interaction with gluons

Due to the binding between heavy quark and antiquark, the states of heavy quarkonium are relatively compact in the standard hadronic scale set by Λ_{QCD} . For this reason the interaction of quarkonium with gluonic fields inside a light meson can be considered in terms of multipole expansion [12, 13]. The leading nonrelativistic (in the heavy quarks) contribution to the van der Waals type interaction then arises in the second order in the chromoelectric dipole term, and can be described by the effective Hamiltonian [14, 15]

$$H_{\text{eff}} = -\frac{1}{2} \alpha^{(Q\bar{Q})} E_i^a E_i^a, \quad (1)$$

where \vec{E}^a is the chromoelectric field and $\alpha^{(Q\bar{Q})}$ is the chromo-polarizability, depending on particular state of the $(Q\bar{Q})$ system, in complete analogy with the description in terms of polarizability of the interaction of atoms with long-wave electric field.

The values of the chromo-polarizability for charmonium and bottomonium levels below the open flavor threshold are all real and positive [15, 16]. The numerical values are so far known only for the off-diagonal polarizability, describing the observed $\pi\pi$ transitions in charmonium and bottomonium. The ‘reference’ values are [15] $\alpha^{(J/\psi\psi')} \approx 2 \text{ GeV}^{-3}$ in charmonium and $\alpha^{(\Upsilon\Upsilon')} \approx 0.6 \text{ GeV}^{-3}$ in bottomonium. It is fully expected [15, 16] that proper diagonal values of the chromo-polarizability in both systems are larger than these ‘reference’ values, especially for excited states. The quoted values correspond to the normalization

convention for the gluonic field strength with the QCD coupling g included in the field strength $F_{\mu\nu}^a$.

In what follows we consider the interaction (1) with the operator $\vec{E}^a \cdot \vec{E}^a$ replaced by $-(F_{\mu\nu}^a)^2/2 = \vec{E}^a \cdot \vec{E}^a - \vec{B}^a \cdot \vec{B}^a$, with \vec{B}^a being the chromomagnetic field. In doing so we consider attraction of quarkonium into gluonic medium which is weaker by the contribution of the manifestly sign-definite term $\vec{B}^a \cdot \vec{B}^a$ than the actual interaction described by the Hamiltonian (1). This clearly leads to a conservative treatment of the problem of existence of bound states [16, 5]. Furthermore, the operator $(F_{\mu\nu}^a)^2$ is related to the trace of the stress tensor θ_μ^μ in QCD with (three) light quarks in the chiral limit by the well known anomaly relation:

$$\theta_\mu^\mu = \frac{\beta(g^2)}{4g^4} (F_{\mu\nu}^a)^2 \approx -\frac{9}{32\pi^2} (F_{\mu\nu}^a)^2, \quad (2)$$

where we replace the QCD beta function by its first one-loop term for definiteness of subsequent numerical estimates, while our general conclusion about existence of bound states does not depend on this replacement. Therefore the form of the van der Waals interaction of heavy quarkonium with light hadronic matter that we consider here is

$$H_W = -C \theta_\mu^\mu \quad (3)$$

with $C = (8\pi^2/9) \alpha^{(Q\bar{Q})} [1 + O(g^2)]$.

We consider the limit of large M_Q so that the kinetic energy of the motion of heavy quarkonium is totally neglected, and we choose the position of quarkonium (in the three dimensional space) at the origin $\vec{x} = 0$. In a holographic model heavy quarkonium is localized at $z = 0$, so that the interaction (3) corresponds to a localized at $(z = 0, \vec{x} = 0)$ source of dilaton.

3 Holographic interaction of light mesons with heavy quarkonium

In a holographic soft wall model [7] an orbitally excited light meson with spin S is described by normalizable solutions $\phi(z, x^\mu)$ of the five-dimensional equation:

$$\left(\partial_\mu \partial^\mu - \frac{\partial^2}{\partial z^2} + z^2 + 2S - 2 + \frac{S^2 - 1/4}{z^2} \right) \phi(z, x) = 0. \quad (4)$$

Upon substitution of a four-dimensional plane wave $\phi(z, x) = e^{-ip \cdot x} \psi(z)$ this results in the eigenvalue problem for the masses $m^2 = p^2$ of the mesons:

$$\left(-\frac{d^2}{dz^2} + z^2 + 2S - 2 + \frac{S^2 - 1/4}{z^2} \right) \psi_n(z) = m_n^2 \psi_n(z) , \quad (5)$$

where n is the excitation number at a given S . The spectrum of m_n^2 is then directly found from the obvious relation of this problem to a two-dimensional harmonic oscillator, and is given by [7]

$$m_n^2 = 4(n + S) , \quad (6)$$

thus reproducing linear Regge trajectories.

It can be mentioned that here dimensionless units are used, corresponding to the coefficient of the “soft wall” term z^2 equal to one. In normal units the energy scale is set by a coefficient σ proportional to the string tension.

In the presence of a static source of the dilaton field located at $(z = 0, \vec{x} = 0)$ the equation (4) is generally no longer translationally invariant in \vec{x} , and the problem becomes a (3+1) dimensional eigenvalue problem for the energy ω :

$$\left[-\Delta_x - \frac{\partial^2}{\partial z^2} + z^2 + 2S - 2 + \frac{S^2 - 1/4}{z^2} + V(z, \vec{x}) \right] \chi_n(z, \vec{x}) = \omega_n^2 \chi_n(z, \vec{x}) , \quad (7)$$

where Δ_x is the three-dimensional Laplacian and $V(z, \vec{x})$ is the potential resulting from the dilaton propagation from the source and interaction with the light-hadron string:

$$V(z, \vec{x}) = g(z) D(z, \vec{x}) \eta , \quad (8)$$

with $D(z, \vec{x})$ being the dilaton propagator from the boundary ($z = 0, \vec{x} = 0$) to the bulk (z, \vec{x}), integrated over time (static source), η is the strength of the source determined by Eq.(3), and $g(z)$ is the vertex for the dilaton-string interaction. The z dependence of the latter vertex arises due to the assumed in Eq.(4) z dependent conformal symmetry breaking term z^2 .

We will show that for the purpose of present discussion there is in fact no need to determine each of the factors in Eq.(8) separately, and that the potential $V(z, \vec{x})$ can be found using the normalization condition for the stress tensor and the fact that the propagation in the bulk of the (conformal dimension $\Delta = 4$) dilaton field from a static source is described by the operator

$$-\Delta_x - \frac{\partial^2}{\partial z^2} + z^2 + 2 + \frac{15}{4z^2} . \quad (9)$$

The z part of this operator corresponds to the motion of two-dimensional harmonic oscillator with angular momentum $j = 2$, and the spatial (\vec{x}) part corresponds to a free motion. Thus one can readily find the propagator as an integral over the proper time τ of the corresponding evolution kernel [17] and write

$$V(z, \vec{x}) = f(z) \int_0^\infty d\tau \left(\frac{1}{2\pi\tau} \right)^{3/2} \exp\left(-\frac{x^2}{2\tau}\right) \frac{e^{-\tau}}{\sinh^3 \tau} \exp\left(-\frac{z^2}{2} \frac{\cosh \tau}{\sinh \tau}\right). \quad (10)$$

The function $f(z)$ here includes the vertex factor $g(z)$ and the source strength η from Eq.(8) as well as normalization and z -dependent metric factors in the propagator $D(z, \vec{x})$.

The function $f(z)$ can be found by comparing the matrix elements of the Hamiltonian (3) and the potential (10) over the plane-wave in x solutions of the equation (4). Indeed for *any* eigenstate in Eq.(5) the normalization of the trace of the stress tensor requires

$$\langle \phi_n(z, x) | \theta_\mu^\mu | \phi_n(z, x) \rangle = 2 m_n^2 \langle \phi_n(z, x) | \phi_n(z, x) \rangle, \quad (11)$$

which determines the average of the interaction (3) over those states. On the other hand, the average of the potential over the same states is given by its integral over \vec{x} :

$$\langle \phi_n(z, x) | V(z, \vec{x}) | \phi_n(z, x) \rangle = \langle \psi_n(z) | \int V(z, \vec{x}) d^3x | \psi_n(z) \rangle = \langle \psi_n(z) | f(z) \frac{4 e^{-z^2/2}}{z^4} | \psi_n(z) \rangle, \quad (12)$$

where in the latter expression the result of explicit calculation of the integral over \vec{x} of the expression from Eq.(10) is used. The latter average corresponds to the condition (11) for *any* eigenstate² if the averaged in the last expression in Eq.(12) function of z is $4(z^2 + S - 1)$. Taking into account also the factor $-C$ in the Hamiltonian (3) one can finally write

$$V(z, \vec{x}) = -c(z^2 + S - 1) z^4 \int_0^\infty d\tau \left(\frac{1}{2\pi\tau} \right)^{3/2} \exp\left(-\frac{x^2}{2\tau}\right) \frac{e^{-\tau}}{\sinh^3 \tau} \exp\left(-\frac{z^2}{2} \frac{e^{-\tau}}{\sinh \tau}\right) \quad (13)$$

with $c = C \sigma^{3/2}$ being the dimensionless value of C .

4 Binding of excited mesons to heavy quarkonium

The problem of existence of bound states between the heavy quarkonium and light mesons is thus reduced to the problem of existence of localized in \vec{x} solutions of the eigenvalue

²In the two-dimensional analog of the z equation (5) this is the virial theorem for harmonic oscillator: the average potential energy is one half of the total.

equation (7). General physical arguments suggest [5] that such solutions exist at a given c and sufficiently large excitation in n or/and S . In practice a full analysis in z and x of states excited in n is significantly complicated by the presence of multitude of lower energy states with lower n and nonzero spatial momentum p . For this reason we analyze here in some detail the excitation of the light meson in the spin S while keeping $n = 0$. The treatment of such excitations is greatly simplified due to the fact that bound state in question is the ground state in the ‘radial’ equation (7) at a given S , so that the corresponding eigenfunction $\chi_0(z, x)$ has no zeros. We thus use a straightforward variational procedure to establish existence of bound states at any nonzero c , provided that S is sufficiently large, and then find numerically the onset of the binding as a function of the coefficient c at few low values of S .

For a variational treatment of the binding problem we choose the *Ansatz* for the eigenfunction in a factorized form

$$\chi_0(z, x) = z^{S+\frac{1}{2}} e^{-z^2/2} \xi(x) , \quad (14)$$

where the z dependence is that of the ground-state wave function of the z equation (5) at a given S . Substituting this variational function in Eq.(7) one finds that the upper bound for the energy of the ground state of the system is then given by the eigenvalue ω found from the x equation

$$[-\Delta_x + U(x)] \xi(x) = (\omega^2 - 4S) \xi(x) , \quad (15)$$

with the effective potential

$$U(x) = -8c(S+1)(S+2) \int_0^\infty d\tau \frac{(S+3)(1-e^{-2\tau}) + S-1}{(2\pi\tau)^{3/2}} \exp\left(-\frac{x^2}{2\tau}\right) (1-e^{-2\tau})^S e^{-4\tau} . \quad (16)$$

Considering large S one can readily see that the factor $(1-e^{-2\tau})^S$ effectively cuts off the contribution of values of τ above $\tau_0 \propto 1/\ln S$, so that the range of the potential behaves as $X_0 \propto 1/\sqrt{\ln S}$. On the other hand the integral over \vec{x} of the potential is proportional to $1/S$, which is weaker than the overall factor $\propto S^3$ in front of and in the integral in Eq.(16). Therefore a bound state exists at sufficiently large S for any nonzero value of the coupling strength c , which is necessarily positive, due to the positivity of the chromo-polarizability.

The results of such variational treatment of the problem of binding are confirmed by an explicit numerical solution of the two dimensional (in z and $|\vec{x}|$) Schrödinger equation with the potential (13). For a fixed S we find that a localized in x ground state solution

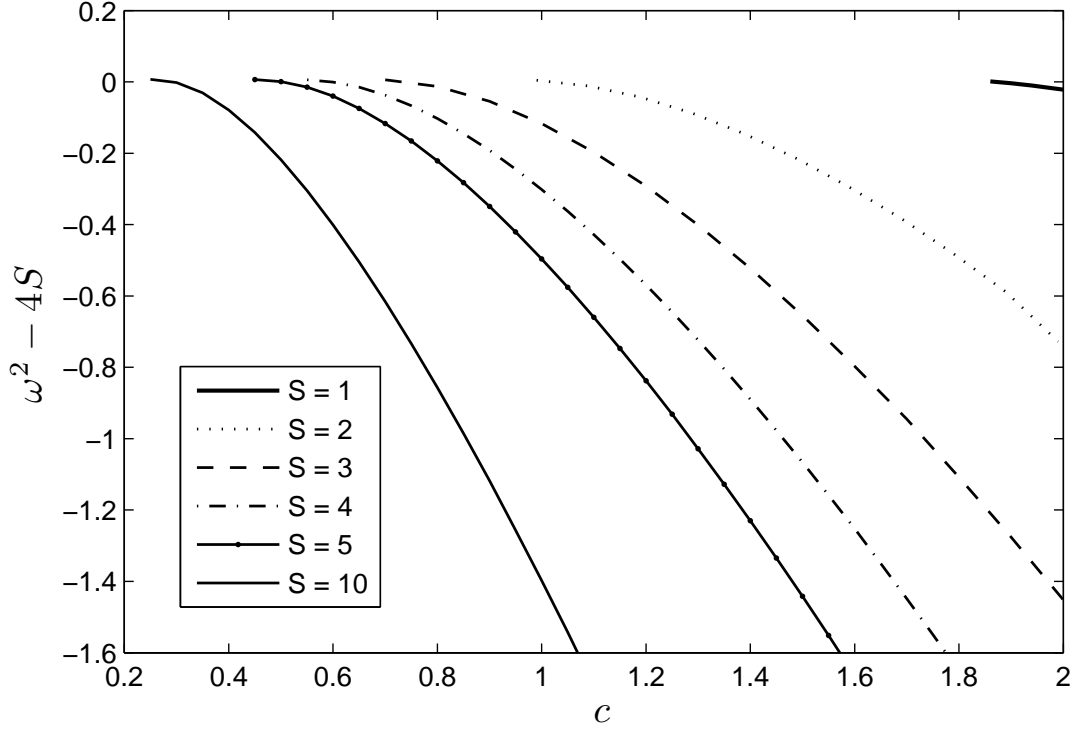


Figure 1: The values of the ground state ‘binding energy’ $\omega^2 - 4S$ as a function of the coefficient c in Eq.(13) for few values of S .

appears starting from a critical (S dependent) value of c . The behavior of the ‘binding energy’ $\omega^2 - 4S$ as a function of c is shown in Fig.1 for few values of S . One can readily see that the critical value of c decreases with S .

Phenomenological applicability of our conclusions to actual charmonium or bottomonium critically depends on the values of the chromo-polarizability for specific quarkonium resonances. Using the estimate for the scale factor σ as [7] $\sigma \approx m_\rho^2/4 \approx 0.15 \text{ GeV}^2$ we find for the ‘reference’ value of $\alpha^{J/\psi\psi'}$: $c \approx 1.0$. According to the data presented in Fig.1 such value of c would correspond to the appearance of binding at $S > 2$. However, as already mentioned, the diagonal chromo-polarizability of the $\psi(2S)$ resonance is likely to be somewhat larger, so that bound hadro-charmonium states may exist already at $S = 2$.

5 Hado-quarkonium decay into pairs of heavy flavored mesons

The notion of hadro-quarkonium necessarily assumes that the heavy quark and antiquark stay bound together as quarkonium inside the host light-matter resonance. In other words the forces from the light quarks do not split the heavy quarkonium, which would result in the decay of the whole system into final states with pairs of heavy flavored mesons, which decays are conspicuously suppressed [4] for the discussed Y and Z resonances. The problem of dissociation of the heavy quarkonium can in fact be treated, to an extent, in the limit of large mass M_Q , in a semiclassical approximation.

Generally the problem can be formulated as that of a reconnection of strings in terms of a holographic correspondence, using, instead of the braneless soft wall model, the underlying scheme with the flavor branes localized in the radial coordinate (for a recent review see e.g. Ref. [18]). The position of the flavor brane in the radial direction is fixed by the quark mass of the corresponding flavor. The mesons in this approach correspond to the open strings with ends on the flavor branes. The ends can be on the same flavor brane, in which case the meson involves quarks of one flavor, or on the different flavor branes which corresponds to a meson built from different types of quarks. Any such model involves a horizon in the radial coordinate which reflects the emergent QCD scale in the holographic picture. The flavor brane corresponding to the light meson is close to the horizon while the brane corresponding to the heavy flavor is far from it.

A bound state of hadro-quarkonium thus involves two strings. One string with the ends on the light-flavor brane(s) and the other on the heavy-flavor one. The decay into states with open heavy flavor mesons corresponds to the reconnection of the strings into the configuration with two open strings both connecting the heavy-flavor brane with the light-flavor one(s). This process is somewhat similar to the previously considered decay of a single excited meson [11] via a reconnection of open string ends, which has been interpreted as a Schwinger type pair production and treated semiclassically. The discussed here reconnection of the strings between light and heavy flavor branes can similarly be traced to an *induced* production of heavy quark pair near the threshold, which approach will be presented elsewhere, and which can be useful in a more elaborate than here treatment.

The leading at large M_Q behavior of the probability of heavy quarkonium dissociation can be evaluated by a simple nonrelativistic quantum-mechanical consideration, using a potential

model for quarkonium combined with a stringy picture at larger separation between quarks. Indeed a potential $V(r)$ between heavy quark and antiquark in quarkonium generically is assumed to be of the form [1, 2]:

$$V_Q(r) = V_c(r) + \sigma r , \quad (17)$$

where the linear term is associated with the stringy behavior at long distances, while the term V_c is the short-distance ‘perturbative’ part, e.g. in the one-gluon exchange picture this is a Coulomb-like potential $-a/r$. In the hadro-quarkonium configuration the total energy of the string is $\sigma (R+r)$, where r is the distance between the heavy quark and the antiquark, and R is the length of the light-meson string. When the string reconnects between heavy and light quarks, there is no string at all between the heavy quark and antiquark, so that the minimal energy of the reconnected configuration is $\sigma (R-r)$. Therefore the dependence of the minimal (over the relative orientations) energy on r becomes in fact given by

$$V_{HQ}(r) = V_c(r) - \sigma r , \quad (18)$$

and this effective potential presents a barrier, tunneling through which results in dissociation of the heavy quarkonium. The tunneling rate then can be estimated as

$$\Gamma \propto \exp \left(-2 \int |p| dr \right) \sim \exp \left(-A \sqrt{\frac{M_Q}{\Lambda_{QCD}}} \right) , \quad (19)$$

where the numerical constant A critically depends on the presently unknown position of the energy of the quarkonium below the top of the barrier. In a generic case of this energy gap being of order Λ_{QCD} this coefficient is parametrically of order one, and the rate is strongly suppressed for heavy quarks.

6 Summary

To summarize. We have considered the van der Waals type interaction between a heavy quarkonium and an excited light meson within a soft wall holographic model of QCD. In this model the heavy quarkonium acts as a source of the dilaton field localized at the boundary. The interaction of the light matter with the dilaton field is then described by a 5D equation with the potential derived from the dilaton propagation from the boundary to the bulk. We have shown by a variational estimate that such interaction necessarily results in bound

hadro-quarkonium states localized near the source at sufficiently high spin excitation of the light meson. We have also found by a numerical calculation that bound states arise at low values of the spin already for moderate values of the strength of the source, which values are close to those expected for charmonium phenomenology. Furthermore, it is argued that the hadro-quarkonium resonances are metastable with respect to dissociation into final states with open-flavor heavy mesons in the limit of large heavy quark mass. The behavior of the hadro-quarkonium states argued in the present paper agrees well with the observed properties of some of the Y and Z resonances near the open charm threshold.

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